3. **Postgame (Flow) Analysis: Inventory Buildup on Gameday**

> *"...If you take away the first quarter of Arizona State's game Saturday against USC, well, the Sun Devils might just name a building after you....*
> *If the Sun Devils are going to salvage something this season, they need to start growing up instead of just talking about it."* – AZ Central

> *" ... We're losing to teams that we're better than."* – Brandon Aiyuk (ASU)

Consider the most recent USC vs. ASU game that you attended on Saturday (November 9th, 2019). After watching a horrendous first quarter, your mind wanders and you start thinking about your "exit strategy"– your best bet on how to exit the stadium at the end of the game. You have the following data:

- **Attendance**:
  - Attendance at the Sun Devil Stadium for the game was estimated to be about 40,625.
  - About three-fifths of them use the light rail (public transportation) on College Avenue, one-fifth use buses (Valley Metro) on Fifth St., while the remaining use private transport (at the nearby parking garage).
- **Light Rail**:
  - To get on a train, fans exit the Sun Devil stadium through the gates on the southwest and the southeast sides of the Stadium, walk to the train station, go through the turnstiles at the light rail station, and climb up the stairs to reach the platform.
  - There are three Exit Gates on the southwest side (SWEG 1 –3) and four gates on the southeast side (SEEG 1—4). Each gate has a capacity of 45 per minute.
  - With no additional attendants, the regular turnstiles at the light rail station process 80 people per minute.
  - The light rail station can supplement the turnstiles with additional attendants. Each additional attendant increases capacity by 20 people per minute. However, each attendant is paid a wage $20 per hour for the hours they provide service.
  - Trains pick up passengers at the rate of 120 per minute (they carry approximately 480 people and run every 4 minutes).
  - Suppose:
    - Fans start to head for the light rail at the end of the third quarter (at 5PM) at a rate of 120 per minute.
    - At the end of the game (6PM), fans start to head to the light rail at an increased rate of 175 per minute.
    - The rest of the light rail users enjoy the stadium's atmosphere, its establishments (e.g., gift shops) and leisurely start to head for the light rail at 7:15PM, at a rate of 45 per minute.
- **Private Transport**:

  - To get to the parking garage, fans exit through the same gates (on the southeast and southwest side of the Stadium), walk to the garage, walk up the stairs to their respective levels, and drive through one of the many exits of the garage.
  - Assume that the parking garage has six exits, each exit has a turnstile and each turnstile can handle 10 per minute.
  - Suppose:
    * At the end of the third quarter (at 5PM), fans start to head for their cars, anticipating high traffic at the end of the game, at a rate of 50 per minute.
    * At the end of the game (6PM), the rate increases to 125 per minute, and continues until all fans (who use the private transport) exit the stadium.

- Assume that the only users of the light rail and the parking garage on game day are fans who watched the USC vs ASU game. Assume that each fan who who uses private transport drives alone (no pooled rides).
- Ignore the operations at the bus stop.

Make (and state) any assumptions that you deem necessary.

(a) (3 points) A schematic of this process flow is shown in Figure 1. To begin, assume that there are no additional attendants at the light rail station. Fill in the (eleven) blanks with the corresponding demand rates, durations, and the resource capacities. Using the space below, explicitly show the corresponding calculations.

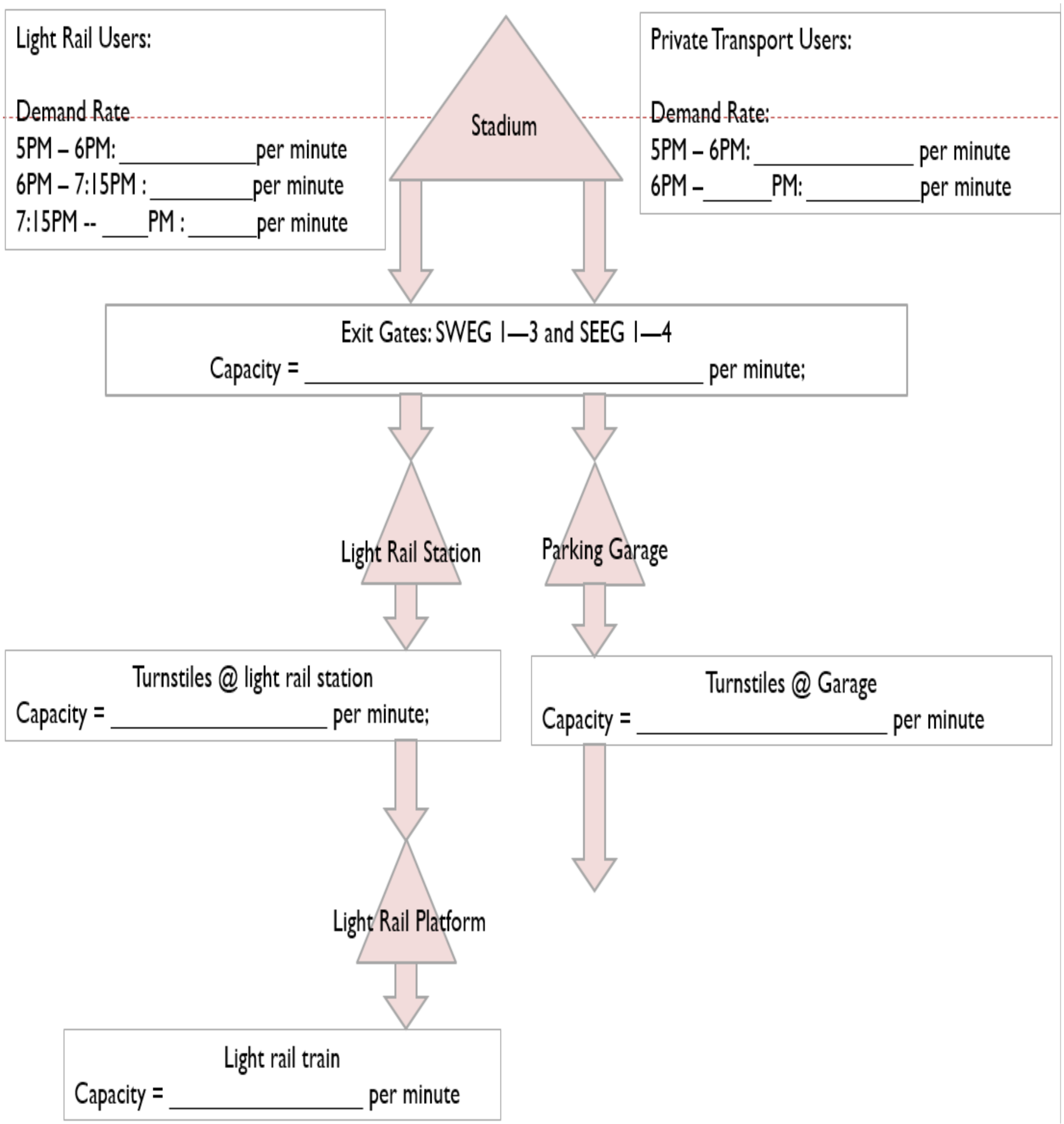

Figure 1: Process Flow Schematic for ASU Fans

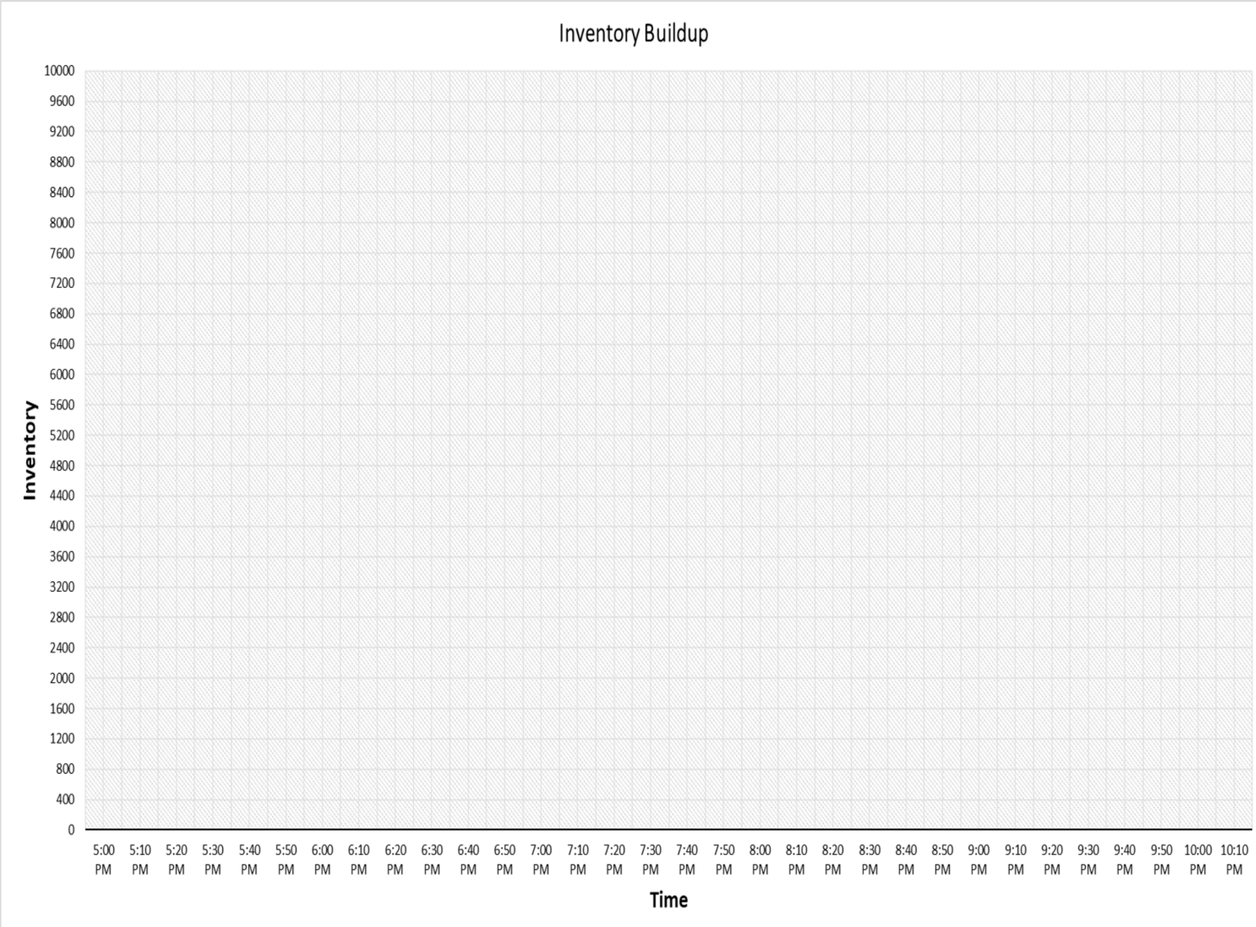

Figure 2: Inventory Buildup Diagram for Light Rail Users

(b) (2.5 points) Consider the light rail users:

- State the bottleneck resource and the buffer where inventory builds up. (Hint: There are three buffers: Stadium, Light Rail Station and the Light Rail Platform)
- Then draw the inventory buildup diagram (at this buffer) in Figure 2. Denote the "slope" (the rate at which inventory builds up and/or depletes) in your diagram.
- Explicitly show the calculations that help you in completing the diagram.

(c) (1.5 points) What is the total user hours spent in waiting by the light rail users? Hints: Area under the curve! Some useful formulae to calculate areas:

- Area of triangle $= \frac{1}{2} \times base \times height$;
- Area of a trapezoid $= \frac{1}{2} \times (a + b) \times height$, where a and b the lengths of the two parallel sides and *height* is the distance between the two parallel sides.

(d) (1.5 points) When does the last light rail user leave the station? (Hint: Use the inventory buildup diagram for the light rail users to identify the time at which the "built-up" inventory is completely exhausted)

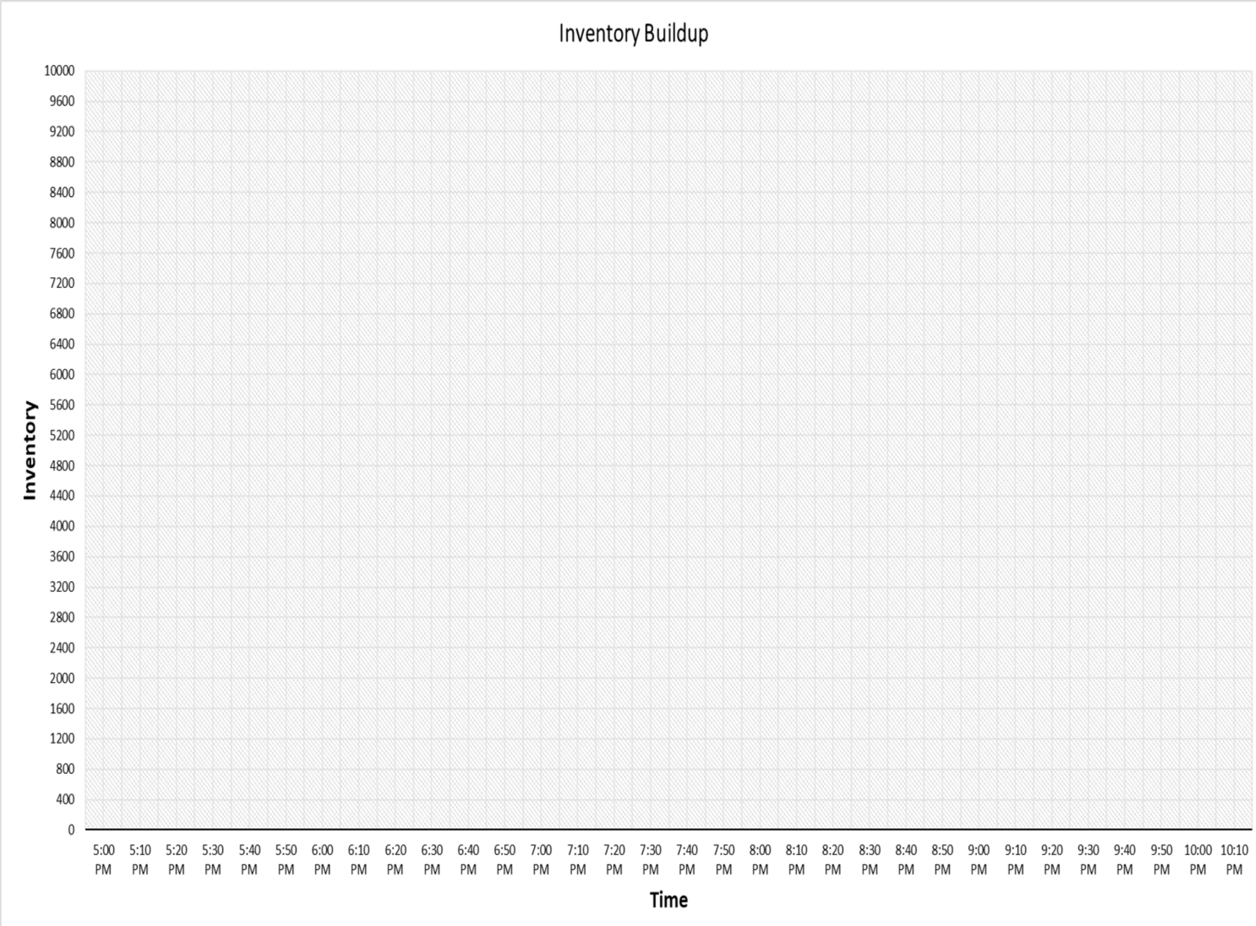

Figure 3: Inventory Buildup Diagram for Light Rail Users with Additional Attendant

(e) (2 points) Suppose you increase the capacity at the light rail station turnstiles with an additional attendant. What is the net reduction in the total user hours spent in waiting at the light rail station due to the additional attendant? (Use the figure if you need to visualize the new process).

(f) (1.5 points) Recall that the additional attendant's wage is \$20 per hour for the hours 5PM until the last light rail user leaves the station. You value each customer-hour at \$0.05. Is adding the additional attendant a profitable idea? Why or why not?

(g) (2 points) Let's switch gears a bit. Consider the arrival process of fans to the stadium before the start of the game. You hav the following data.

- Fans use the same gates (SWEG 1 –3 and SEEG 1–4) to enter the stadium.
- Each gate is manned by a student worker who validates each user's ticket by scanning a QR code using a handheld device.
- The student worker can scan 45 QR codes each minute, and there is no variability in his processing time.
- You observe that each of these seven gates use separate queues for entry.
- Suppose the arrival process is modeled as a Poisson process. Fans arrive at the rate of 280 per minute, and choose one of the seven gates at random.

What is the average waiting time in a queue? What is the average length of each separate queue? Hints:

- Since each gate has a separate queue, the "total" arrival rate is 280 per minute and fans choose a gate at random, the arrival rate at each gate is $\frac{280}{7} = 40$ per minute.
- Use the VUT equation with a single-server to calculate the waiting time at each gate.
- $CV_a = 1$ (Why? Arrivals are modeled as a "Poison" process), $CV_p = 0$ (the student worker can serve "exactly" 45 students a minute).

(h) (2.5 points) Based on what you learnt in SCM 502, you argue that the Sun Devil Stadium should adopt a "pooled" queuing design for entry. What is the (net) reduction in the waiting time due to pooled queues?

(i) (.5 point) Let's conclude: Suppose fans head to one of the many food stalls during halftime at the Stadium. Consider the operations at one such food stall. If a customer spends 7.5 minutes in the queue and the length of the queue is 30 customers, what is the throughput rate of the food stall? (Hint: Use Little's Law)